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SCIENCE IN THE 21ST CENTURY:

CHALLENGES, SOLUTIONS AND DEVELOPMENT STRATEGIES

January 22, 2026

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SOBOLEV FAZOSIDA KVADRATUR FORMULA XATOLIK FUNKSIONALINING NORMASI

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Annotatsiya: *Ushbu maqolada* $W_2^{(2)}(0,1)$ *Sobolev fazosida, ya’ni [0,1] oraliqda ikkinchi tartibli umumlashgan hosilasi kvadrati bilan integrallanuvchi funksiyalar sinfi uchun optimal kvadratur formulani qurish masalasi ko‘rib chiqiladi. Chiziqli uzluksiz funksionalning umumiy ko‘rinishi haqidagi Riss teoremasidan foydalanib, kvadratur formulaning xatolik funksionali uchun ekstremal funksiyani aniqlash masalasi oddiy differensial tenglamalar uchun chegaraviy masalani yechishga keltiriladi. Ushbu chegaraviy masalani yechish orqali kvadratur formulaning xatolik funksionali uchun ekstremal funksiyaning aniq analitik ko‘rinishi olinadi. Bundan tashqari, Sobolev fazosida kvadratur formulaning xatolik funksionali normasining yopiq analitik ifodasi keltirib chiqariladi.*

Kalit so‘zlar: *Kvadratur formula, ekstremal funksiya, xatolik funksionali .*

Kirish. Masalani qo‘yilishi.

Integral ifodalarni sonli yaqinlashtirish masalalari hisoblash matematikasining muhim yo‘nalishlaridan biri bo‘lib, ularning samaradorligi kvadratur formulalarning aniqligi bilan bevosita bog‘liq. Shu sababli, berilgan funksiyalar sinfi uchun optimal kvadratur formulalarni qurish va ularning xatoliklarini qat'iy baholash dolzarb masalalardan hisoblanadi. Sobolev fazosida kvadratur formulaning xatolik funksionali chiziqli uzluksiz funksional sifatida qaraladi va uning tahlilida Riss teoremasidan foydalanish muhim nazariy asos bo‘lib xizmat qiladi. Ushbu yondashuv xatolik funksionalining ekstremal funksiyasini aniqlash masalasini oddiy differensial tenglamalar uchun chegaraviy masalani yechishga keltirish imkonini beradi. Natijada kvadratur formulaning xatolik funksionalining ekstremal funksiyasi hamda uning normasi uchun analitik ifodalar olinadi. Mazkur ishda $W_2^{(2)}(0,1)$ Sobolev fazosida optimal kvadratur formulani qurish, xatolik funksionalining ekstremal funksiyasini topish va uning normasini analitik ko‘rinishda aniqlash masalalari tadqiq etiladi.

Biz quyidagi ko‘rinishda kvadratur formulani qaraymiz [1-2]

$$\int_0^1 \varphi(x)dx \cong \sum_{\beta=0}^{N} k[\beta]\varphi(h\beta) \quad (1)$$

bu yerda $k[\beta]$- (1) kvadratur formulaning hozircha noma'lum koeffitsiyentlari, $h=1/N$, $N-$ natural son, integral ostidagi $\varphi(x)$ funksiya $W_2^{(2)}(0,1)$ fazoga tegishli. Bunda $W_2^{(2)}(0,1)$ fazo bu birinchi tartibli hosilasi absolyut uzluksiz va ikkinchi tartibli hosilasi $L_2(0,1)$ ga tegishli barcha funksiyalar sinfi. Ushbu $W_2^{(2)}(0,1)$ fazo

$$\langle \varphi,\psi \rangle_W = \int_0^1 (\varphi''\psi'' + \varphi'\psi'')dx. \quad (2)$$

skalyar ko‘paytmaga nisbatan Gilbert fazosi bo‘ladi. Bu yerda (2)- skalyar ko‘paytma yordamida norma quyidagicha aniqlanadi

$$\|\varphi\|_{W_2^{(2)}(0,1)} = \sqrt{\langle \varphi,\varphi \rangle} = \sqrt{\int_0^1 \left((\varphi'')^2 + (\varphi')^2 \right) dx} \quad (3)$$

(1)-kvadratur formulaning xatoligi deb

$$\int_0^1 \varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\varphi(h\beta) \quad (4)$$

ayirmaga aytiladi va bu ayirmaga $W_2^{(2)}(0,1)$ fazosida aniqlangan quydagi xatolik funksionali mos keladi

$$\ell(x) = i_{[0,1]}(x) - \sum_{\beta=0}^{N} k[\beta]\delta\left(x - h\beta\right) \ , \tag{5}$$

bu yerda $i_{[0,1]}(x)$-[0,1] kesmaning xarakteristik funksiyasi, $\delta(x)$-Dirakning delta-funksiyasi.

Odatda, $\ell(x)$ funksionalning $\varphi(x)$ funksiyadagi qiymati quyidagicha aniqlanadi

$$\left(\ell,\varphi\right) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx\,. \tag{6}$$

(6) tenglikka asosan, (5) formulani etiborga olib, (4) ayirma haqiqatdan ham, $\ell(x)$ xatolik funksionalining $\varphi(x)$ dagi qiymati ekanligiga ishonch hosil qilamiz, ya'ni

$$\left(\ell,\varphi\right) = \int_{-\infty}^{\infty} \ell(x)\varphi(x)dx = \int_{-\infty}^{\infty}\left(i_{[0,1]}(x) - \sum_{\beta=0}^{N} k[\beta]\delta\left(x - h\beta\right)\right)\cdot\varphi\left(x\right)dx =$$

$$= \int_{-\infty}^{\infty} i_{[0,1]}(x)\varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\delta\left(x - h\beta\right)\cdot\varphi\left(x\right)dx = \int_{0}^{1}\varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\varphi\left(h\beta\right).$$

Demak,

$$(\ell,\varphi) = \int_{0}^{1}\varphi(x)dx - \sum_{\beta=0}^{N} k[\beta]\varphi\left(h\beta\right) \tag{7}$$

(1) kvadratur formulaning (7) xatoligi $W_2^{(2)*}(0,1)$ fazosida chiziqli funksionalni aniqlaydi,

bu yerda $W_2^{(2)*}$ fazo $W_2^{(2)}$ fazoga qo'shma fazo. U holda, Koshi-Shvars tengsizligidan xatolikning absolyut qiymati yuqoridan quyidagicha baholanadi:

$$\left|(\ell,\varphi)\right| \le \left\|\varphi \,|\, W_2^{(2)}(0,1)\right\|\cdot\left\|\ell \,|\, W_2^{(2)*}(0,1)\right\|$$

Bu tengsizlikdan biz (1) kvadratur formulaning (7) xatoligi $\ell(x)$ xatolik funksionalining

$$\left\|\ell \mid W_2^{(2)*}(0,1)\right\| = \sup_{\left\|\varphi|W_2^{(2)}(0,1)\right\|=1} \left|(\ell,\varphi)\right| = \sup_{\varphi,\|\varphi\|\neq 0} \frac{\left|(\ell,\varphi)\right|}{\left\|\varphi \mid W_2^{(2)}(0,1)\right\|}$$

normasi orqali baholanishini xulosa qilamiz.

Yuqoridagi tengsizlikdan ko‘rinadiki, (4) xatolikning absolyut qiymatini yuqoridan baholash uchun, (5) xatolik funksionalining normasini topish talab etiladi. Buning uchun esa mos ekstremal funksiyani aniqlash kerak. Quyida aynan shu ekstremal funksiyani aniqlash masalasini qaraymiz.

Ushbu masalani yechish uchun, exstremal funksiya ta’rifidan foydalanamiz. Ma’lumki, Koshi-Shvarts tengsizligini tenglikka aylantiruvchi $U_\ell(x)$ funksiyaga ekstremal funksiya deyiladi, ya’ni

$$(\ell,U_\ell) = \left\|\ell \middle| W_2^{(2)*}(0,1)\right\| \cdot \left\|U_\ell \middle| W_2^{(2)}(0,1)\right\|. \tag{8}$$

$W_2^{(m)}(0,1)$ Gilbert fazosida chiziqli uzluksiz funksionalning umumiy ko‘rinishi haqidagi Riss teoremasidan foydalanib quyidagini yozamiz.

$$\left\|\ell \middle| W_2^{(2)*}(0,1)\right\| = \left\|U_\ell \middle| W_2^{(2)}(0,1)\right\|. \tag{9}$$

Shuningdek, (8) va (9) dan quyidagi xulosaga ega bo‘lamiz:

$$(\ell,U_\ell) = \left\|\ell \middle| W_2^{(2)*}(0,1)\right\|^2. \tag{10}$$

Boshqa tomondan, xuddi shu teorema bo‘yicha, $W_2^{(2)}(0,1)$ fazoning har qanday $\varphi(x)$ elementi uchun ushbu tenglikni olamiz

$$(\ell,\varphi) = \langle U_\ell,\varphi\rangle,$$

bu yerda

$$<U_\ell,\varphi> = \int_0^1 \left(\frac{d^2\varphi(x)}{dx^2} \cdot \frac{d^2U_\ell(x)}{dx^2} + \frac{d\varphi(x)}{dx} \cdot \frac{dU_\ell(x)}{dx} \right) dx. \tag{11}$$

Aytaylik, $\varphi(x)$ funksiya $W_2^{(2)}(0,1)$ fazoga tegishli cheksiz differensiallanuvchi funksiya bo‘lsin, ya’ni.

$$\varphi(x) \in \overset{\circ}{C}{}^{(\infty)}(0,1).$$

(11) tenglikning o‘ng tomonini bo‘laklab integallab quyidagini hosil qilamiz

$$U_\ell^{(4)}(x) - U_\ell^{(2)}(x) = \ell(x). \qquad (12)$$

$$\left(U_\ell^{(3)}(x) - U_\ell^{(1)}(x)\right)\Big|_{x=0}^{x=1} = 0, \qquad (13)$$

$$U_\ell^{(2)}(x)\Big|_{x=0}^{x=1} = 0. \qquad (14)$$

Teorema 1. *(12)-(14) chegaraviy masalaning yechimi (1) kvadratur formulaning (5) xatolik funksionali ekstremal funksiyasi bo‘lib, quyidagi ko‘rinishga ega.*

$$U_\ell(x) = \ell(x) * \mu_2(x) + b_0,$$

bu yerda

$$\mu_2(x) = \frac{signx}{2}\left(\frac{e^x - e^{-x}}{2} - x\right) \qquad (15)$$

ushbu $\frac{d^4}{dx^4} - \frac{d^2}{dx^2}$ *differensial operatorning fundamental yechimi,*

$$signx = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0, \end{cases}$$ b_0 *-ixtiyoriy o‘zgarmas son.*

1-teorema to‘liq isbotlandi.

Kvadratur formulaning xatolik funksionali normasi.

Endi biz xatolik funksionali normasini hisoblaymiz. Quyidagi teorema o‘rinli.

Teorema 2. Xatolik funksionalning normasining kvadrati quyidagi ko‘rinishda

$$\left\|\ell(x)\middle|W_2^{(2)*}(0,1)\right\|^2 = \left[\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N} k[\beta]k[\gamma]\mu_2(h\beta - h\gamma) - \right.$$

$$\left. -2\sum_{\beta=0}^{N} k[\beta]\int_0^1 \mu_2(x - h\beta)dx + \int_0^1\int_0^1 \mu_2(x-y)dxdy\right]. \qquad (16)$$

Бу ерда $\mu_2(x)$ Grin funksiyasi (15) formula bilan aniqlanadi.

Teoremani isboti 2. $W_2^{(2)}(0,1)$ fazo Gilbert bo‘lganligi sababli, chiziqli funksionalning umumiy shakli bo‘yicha Riss teoremasi bo‘yicha va ekstremal funksiyaning ta’rifini hisobga olgan holda, biz

$$(\ell, U_\ell) = \left\| \ell(x) \middle| W_2^{(2)*}(0,1) \right\| \cdot \left\| U_\ell \middle| W_2^{(2)}(0,1) \right\| = \left\| U_\ell \middle| W_2^{(2)}(0,1) \right\|^2 = \left\| \ell(x) \middle| W_2^{(2)*}(0,1) \right\|^2.$$

demak, 1-teoremani hisobga olsak, biz bor

$$\left\| \ell(x) \middle| W_2^{(2)*}(0,1) \right\|^2 = (\ell, U_\ell) = \int \ell(x) U_\ell(x) dx =$$

$$= \int_{-\infty}^{\infty} \left(i_{[0,1]}(x) - \sum_{\beta=0}^{N} k[\beta] \delta(x - h\beta) \right) \times$$

$$\times \left(\ell(x) * \frac{signx}{2} \left(\frac{e^x - e^{-x}}{2} - x \right) + P_0(x) \right) dx.$$

Tenglikdan (8) foydalanib, biz olamiz

$$\left\| \ell(x) \middle| W_2^{(2)*}(0,1) \right\|^2 = \int_{-\infty}^{\infty} \ell(x) \left(\ell(x) * \frac{signx}{2} \left(\frac{e^x - e^{-x}}{2} - x \right) \right) dx. \tag{17}$$

Birinchidan, biz svyortkasini hisoblaymiz

$$\ell(x) * \frac{signx}{2} \left(\frac{e^x - e^{-x}}{2} - x \right)$$

$$\ell(x) * \frac{signx}{2} \left(\frac{e^x - e^{-x}}{2} - x \right) =$$

$$= \int_{-\infty}^{\infty} \ell(y) * \frac{sign(x-y)}{2} \left(\frac{e^{x-y} - e^{y-x}}{2} - (x-y) \right) dy =$$

$$= \int_{-\infty}^{\infty} \left(i_{[0,1]}(y) - \sum_{\beta=0}^{N} k[\beta] \delta(y - h\beta) \right) * \frac{sign(x-y)}{2} \left(\frac{e^{x-y} - e^{y-x}}{2} - (x-y) \right) dy =$$

$$= \int_0^1 \frac{sign(x-y)}{2} \left(\frac{e^{x-y} - e^{y-x}}{2} - (x-y) \right) dy -$$

$$-\sum_{\beta=0}^{N} k[\beta] \frac{sign(x-h\beta)}{2}\left(\frac{e^{x-h\beta}-e^{h\beta-x}}{2}-(x-y)\right). \qquad (18)$$

Keyin (18) ni (17) ga almashtirib, biz quyidagini hosil qilamiz

$$\left\|\ell(x)\middle|W_2^{(2)*}(0,1)\right\|^2=\int\left(i_{[0,1]}(x)-\sum_{\beta=0}^{N}k[\beta]\delta(x-h\beta)\right)\times$$

$$\times\left(\int_0^1\frac{sign(x-y)}{2}\left(\frac{e^{x-y}-e^{y-x}}{2}-(x-y)\right)dy-\right.$$

$$\left.-\sum_{\gamma=0}^{N}k[\gamma]\frac{sign(x-h\gamma)}{2}\left(\frac{e^{x-h\gamma}-e^{h\gamma-x}}{2}-(x-h\gamma)\right)\right)dx.$$

Shunday qilib, qavslarni ochib ifodani soddalashtirib, biz quyidagi natijani olamiz

$$\left\|\ell(x)\middle|W_2^{(2)*}(0,1)\right\|^2=\left[\sum_{\beta=0}^{N}\sum_{\gamma=0}^{N}k[\beta]k[\gamma]\mu_2(h\beta-h\gamma)-2\sum_{\beta=0}^{N}k[\beta]\int_0^1\mu_2(x-h\beta)dx+\right.$$

$$\left.+\int_0^1\int_0^1\mu_2(x-y)dxdy\right]. \qquad (19)$$

Bu yerdan, biz darhol (16) olamiz. 2-teorema to‘liq isbotlangan.

XULOSA

Shunday qilib, biz ushbu ishda $W_2^{(2)}(0,1)$ fazoda optimal kvadratura formulasini qurishni ko‘rib chiqdik. Bundan tashqari, kvadratura formulalarining ekstremal funksiyasini, xatolik funksionalining normasini ko‘rinishini topdik .

Foydalanilgan adabiyotlar

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