SOLVING LINEAR PROGRAMMING PROBLEMS IN OPTIMIZATION OF ECONOMIC PROCESSES USING THE SIMPLEX METHOD

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Abstract: This article describes the essence, algorithm, and practical application of the simplex method , one of the most effective and universal methods used to solve linear programming problems . In most cases, the problem of finding an optimal solution in linear programming problems cannot be solved using geometric methods, especially when the number of unknowns is more than three. Therefore, a special algorithmic approach is required to solve large-scale problems. The article presents the theoretical foundations of the simplex method, and explains in detail the process of transforming the problem into a canonical form, selecting an initial base solution, and achieving an optimal solution through successive iterations using an example. Obtained results of the simplex method efficiency and practical issues in solution convenience shows .

Key words : linear programming , optimal solution , support solution , simplex method , canonical appearance , mathematics modeling .

1. Introduction

Linear programming issues and them solution methods optimization theory important from directions one This is in the field take visited research as a result many effective algorithms working issued are , they are between the most wide widespread from methods one **simplex method** is . Linear programming issues theoretical in terms of justification and optimal solutions find issues scientific in literature wide covered [1, 2].

Simples method originally G. Danzig by offer done this is method linear restrictions with given optimal solution to problems support solutions sequence improve through to find is based on [3]. Researchers by take visited scientific affairs this shows that the simplex method big dimensional linear programming issues in solution high to efficiency has and calculation processes simplification opportunity gives [4].

One row scientific in sources linear programming issues geometric and algorithmic methods using solution opportunities compared . This in research unknowns number three more than was in cases geometric of the method application limitedness based on given and such in cases of simples method column aspects showing [5]. Also , the issue canonical to look to bring , artificial variables input and basis choice processes in detail analysis made .

Modern in research linear programming issues computer technologies using solution also big on issues attention Scientific simplex method in literature software in supplies done increase , calculation accuracy and speed according to advantages [6]. This is the result of the simplex method . not only theoretically , maybe practical also important importance has that shows .

Literature analysis this shows that linear programming issues the simples method in learning theoretical the basics practical examples with harmonious without statement to grow current is considered . This in the article exactly this approach based on the simplex method essence and algorithm is illuminated .

2. Materials and Methods

When solving linear programming problems (LPPs), it is necessary to find the optimal solution among the base solutions. If the number of unknowns is more than three, we cannot use the geometric method to find the base solution. Therefore, we need to choose a universal method for solving such problems, regardless of their size. One of such methods is the simplex method, in other words, this method can also be called a method of improving plans. The essence of the simplex method is that we first construct its initial base solution plan. As a result of checking the initial condition, we obtain the next improved solution plan. We continue this process until we obtain the optimal solution plan. At each step, we obtain another base solution to the problem. If the number of base solutions is equal to the number of solutions at the vertices of the polygon, then the number of solutions in the simplex method is also equal to the number of its steps. To solve linear programming problems in the simplex method, it is necessary to reduce it to canonical form, that is, the inequalities in all constraint conditions must be reduced to the form of equations. We achieve this by introducing artificial unknowns on the left side of the inequality.

We consider this process in the following linear programming problem:

$$\begin{cases} \sum_{j=1}^{n} a_{ij}x_j \le b_i, \quad i=1,2,\dots,m, & (1) \\ x_j \ge 0, \quad j=1,2,\dots,n, & (2) \\ L(x)=\sum_{j=1}^{n} c_j x_j \to \max. & (3) \end{cases}$$

First, to transform inequality (1) into an equation, we substitute artificial variables x_{n+1} , x_{n+2} ,..., x_{n+m} , on the left side of the inequality. we add the unknowns, in the objective function for these unknowns $c_{n+1} = c_{n+2} = \ldots = c_{n+m} = 0$ will be equal to That is, the cost of artificial variables will be zero. After that, we write problems 1-3 as follows:

$$\begin{cases} \sum_{j=1}^{n} a_{ij}x_j + x_{n+i} = b_i, \quad i = 1, 2, \ldots, m, & (4) \\ x_j \geq 0, \quad j = 1, 2, \ldots, n+m, & (5) \\ L = \sum_{j=1}^{n+m} c_j x_j \to \max. & (6) \end{cases}$$

It can be seen that the values of x $L(x)_{n+1}$, x_{n+2} ,…,x_{n+m} , do not affect the objective function, since $c_{n+i} = 0$; $i = 1, 2, \ldots, m$.

The matrix representation of Linear Programming Problem 4-6 can be written in short form as:

$$\begin{cases} A \cdot X = B & (7) \\ C \cdot X \to \max. & (8) \end{cases}$$

Here a $A = (a_{ij})$. $A(n \times (n+m))$ – rectangular matrix. $C = (c_1, c_2, \ldots, c_{n+m})$ – row matrix,

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n+m} \end{pmatrix} - \text{column matrix}, \quad B = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix} - \text{column matrix}.$$

Now let's consider the essence of the simplex method. To do this, we first need to choose the basic variables, the basic variables must be equal to the number of reserves m . The column matrix A formed from the basic variables must be a unit matrix. The values of the basic variables are taken from the values on the right side of the equation. If the system does not satisfy the conditions, then it is necessary to reduce it to this form. After that, we proceed to the construction of the first simplex table, which will look like this:

Table 1 .

I	C_j \ C_{ki}		C_1	C_2	C_3	…	C_{n+m-1}	C_{n+m}		
		basis	A_1	A_2	A_3	…	A_{n+m-1}	A_{n+m}	B	δ_i
1	C_{k1}	X_{k1}	a_{11}	a_{12}	a_{13}	…	$a_{1\,n+m-1}$	$a_{1\,n+m}$	b_1	

2	C_{k2}	X_{k2}	a_{21}	a_{22}	a_{23}	…	$a_{2\ n+m-1}$	$a_{2\ n+m}$	b_2	
…	…	…	…	…	…	…	….	…	…	
m -1	$C_{k_{m-1}}$	$X_{k_{m-1}}$	$a_{m-1\ 1}$	$a_{m-1\ 2}$	$a_{m-1\ 3}$	…	$a_{m-1\ n+m}$	$a_{m-1\ n+m}$	b_{m-1}	
m	C_{k_m}	X_{km}	$a_{m\ 1}$	$a_{m\ 2}$	$a_{m\ 3}$	…	$a_{m\ n+m-1}$	$a_{m\ n+m}$	b_m	
		$j\Delta$								

following are selected as the baseline variables : $x_{k_1}, x_{k_2}, ..., x_{k_m}$ their values of the equation right on the side $b_1, b_2, ..., b_m$.to values equal because Let's take . In the first table, the values in the column of the basic variables must be equal to one (one of the remaining ones must be equal to zero). All other non-basic variables are taken to be equal to zero.

We can interpret the criterion for an optimal solution that corresponds to the table as follows: In each column, from A_{1} to A_{n+m} calculations are made as follows:

$$\Delta_j = \sum_{i=1}^{m} a_{ij} \cdot c_{k_i} - c_j, \qquad j = 1, 2, ..., n, n+1, n+m. \qquad (9)$$

If the table Δ_j has no negative values in the row numbers, then this table is an optimal plan and the calculation is stopped. If the table Δ_j has negative values in the row numbers, this plan is not optimal, and the calculation is continued in this case. The search for the optimal plan is continued step by step. Δ_j The smallest column in the row is selected from among the negative values, the selected Δ_j column is taken as the solver , marked with (→), and the element to be solved is selected using the following relation:

$$\delta_i = \frac{b_i}{a_{il}}, \qquad i = 1, 2, ..., m, \qquad (10)$$

Here l –means that the solver is dominant . If an array element is negative $a_{il} \leq 0$ will be skipped .δ_i the small value of and of the corresponding S series δ_S element is chosen as the determinant. That is, since the element a_{Sl} is located at the intersection of the determinant column and the determinant row, it is taken as the determinant element. After that, we proceed to fill in the next simplex table. This process is carried out similarly to the method of successive elimination of unknowns for solving linear algebraic equations. First, we divide the elements in the determinant row by the determinant element. Instead of the determinant element, we obtain one. Then, we multiply the numbers in the determinant row by such a suitable number and add them to the corresponding numbers in the other row so that the numbers remaining in our determinant column become zero. In the case we are looking at, the multipliers are equal to the following $a_{il}, i = 1, 2, ..., s-1, s+1, ..., m.$. Then we move on to the second simplex table, and in this table we repeat the operations performed above. This process is continued until we achieve the optimal design result.

3. Results and Discussion

Now we will consider the solution of the following problem using the simplex method.

$$\begin{cases} 0,1x_1 + 0,3x_2 \le 30 \\ 0,5x_1 + 0,2x_2 \le 45 \\ 0,1x_1 + 0,1x_2 \le 12 \\ x_1 \ge 0;\ x_2 \ge 0 \\ L(x_1, x_2) = 1000x_1 + 1400x_2 \to \max. \end{cases}$$

We can bring this problem to the canonical form, that is, x_3, x_4, x_5. we introduce artificial variables into the left-hand side of the inequality

$$\begin{cases} 0,1x_1 + 0,3x_2 + x_3 = 30 \\ 0,5x_1 + 0,2x_2 + x_4 = 45 \\ 0,1x_1 + 0,1x_2 + x_5 = 12 \\ L(x_1, x_2) = 1000x_1 + 1400x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5 \to \max. \end{cases}$$

Let's make the first simplex table for this equation:

Table 2

I	C_j \ C_{ki}		*1000*	*1400*	*0*	*0*	*0*			
		Some name	A_1	A_2	A_3	A_4	A_5	*B*	δ_i	
1	*0*	X_3	*0.1*	*0.3*	*1*	*0*	*0*	*30*	*100*	←
2	*0*	X_4	*0.5*	*0.2*	*0*	*1*	*0*	*45*	*225*	
3	*0*	X_5	*0.1*	*0.1*	*0*	*0*	*1*	*12*	*120*	
		Δ	*-1000*	*-1400* →	*0*	*0*	*0*			

We fill in the first table directly without calculations according to the condition of the problem. Here, the basic variables x_3, x_4, x_5 are also taken from the problem statement. In the table, the corresponding values of these variables are indicated by one. The column with the values of the basic variables is also indicated by zero, i.e. $s_3 = s_4 = s_5 = 0$. Now $\Delta_j = c_k \cdot A_j - c_j$ we calculate $\Delta_j = c_k \cdot A_j - c_j$

$\Delta_1 = 0 \cdot 0,1 + 0 \cdot 0,5 + 0 \cdot 0,1 - 1000 = -1000$

$\Delta_2 = 0 \cdot 0,3 + 0 \cdot 0,2 + 0 \cdot 0,1 - 1400 = -1400.$

This The decisive element is in the second column. It is marked with an arrow in the table. Since there are negative values in the row in the table Δ_j, the $(x_1 = 0;\ x_2 = 0;\ x_3 = 30;\ x_4 = 45;\ x_5 = 12)$ solution plan in this row is not optimal. δ_i We determine the values of the column in the table using the following formula . $\delta_i = \dfrac{b_i}{a_{i2}},\quad i = 1,2,3.$ The smallest δ_i value of the results found is 100, therefore, the decisive element is in the first row. We mark this row in the table with an arrow. We mark the decisive element as 0.3. We begin to fill in the second simplex table. The size of this table will be the same as the previous one. We begin to fill in the table from the decisive row. We divide all the numbers in the row by 0.3.

Table 3 .

I	C_j \ C_{ki}		*1000*	*1400*	*0*	*0*	*0*		
		base	A_1	A_2	A_3	A_4	A_5	*B*	δ_i
1	*1400*	X_2	*1/3*	*1*	*10/3*	*0*	*0*	*100*	*300*
2	*0*	X_4	*13/30*	*0*	*-2/3*	*1*	*0*	*25*	*16000/13*
3	*0*	X_5	*2/30*	*0*	*-1/3*	*0*	*1*	*2*	*30* ←
		Δ_j	*-1600/3*	*0*	*14000/3*	*0*	*0*		

The remaining rows in the table are filled in using the above calculations. We multiply the newly formed row by 0.2, subtract the elements of the second row, and write them in this row. We multiply the third row by 0.1 and perform the above operation. This table plan will be equal to $x_2 = 100;\ x_4 = 25;\ x_5 = 2;\ x_1 = 0;\ x_3 = 0.$

We check the found plan for optimality. Δ_j we calculate :

$$\Delta_1 = 1400 \cdot \frac{1}{3} - 1000 = -\frac{1600}{3};\quad \Delta_2 = 1400 \cdot 1 - 1400 = 0;$$

$$\Delta_3 = 1400 \cdot \frac{10}{3} - 0 = \frac{1400}{3}.$$

Since there are negative values in this row, we go to the third simplex table. Δ_j As before, we select the solving series and write this series dividing into the solving element.

Table 4 .

I	C_j / C_{ki}		*1000*	*1400*	*0*	*0*	*0*	
		base	A_1	A_2	A_3	A_4	A_5	b_i
1	*1400*	X_2	*0*	*1*	*5*	*0*	*-5*	*90*
2	*0*	X_4	*0*	*0*	*3*	*1*	*-6.5*	*12*
3	*1000*	X_1	*1*	*0*	*-5*	*0*	*15*	*30* ←
			0	*0*	*2000*	*0*	*8000*	

All rows $\Delta_j \geq 0$.in this table are positive Δ_j Therefore, the found plan will be optimal. Thus, the plan found in the table will be optimal. In the last table plan, our answer is: $x_1 = 30;\ x_2 = 90;\ x_3 = 0;\ x_4 = 8;\ x_5 = 0$. In the simplex method, we obtain a base solution at each step, and gradually approach the optimal solution. We start at the point O(0,0) in the first step, i.e. $x_1 = 0;\ x_2 = 0$. In the second step, we move to point A (0;100). In the third step, we move to point V(30,90). This corresponds to the simplex idea.

It should be noted that in the simplex method it is possible to find the solution of both the given and the dual problem at the same time. In the last column of the simplex table b_i, we get the value of the base variables. In our table, these values $x_1 = 30;\ x_2 = 90;\ x_4 = 12$ are equal to The last row Δ_j contains the solutions of the dual problem under artificial basis unknowns. In our example, these values $\Delta_3 = 2000;\ \Delta_4 = 0;\ \ \Delta_5 = 8000$ are equal to From this we $y_1 = 2000;\ y_2 = 0;\ y_3 = 8000$ can see that we. We calculate the value for this score Q :

$$Q = 30y_1 + 45y_2 + 12y_3 \to \min,$$

$$Q_{\min} = 30 \cdot 2000 + 45 \cdot 0 + 12 \cdot 8000 = 156000.$$

If we compare this solution with the primary solution, we see the same thing :

$$Q_{\min} = L_{\min} = 156000.$$

This is consistent with duality theory.

4. Conclusion

This article examines in detail the theoretical foundations and practical application of one of the most effective algorithmic methods used in solving linear programming problems - **the simplex method** . During the study, the process of canonicalizing the linear programming problem, selecting an initial base solution, and achieving an optimal solution using successive simplex tables was consistently analyzed.
The analyses conducted have shown that, although the ability to solve linear programming problems with a large number of unknowns using geometric methods is limited, the simplex method provides stable and reliable results even in such cases. Each one in iteration support solutions step by step getting better progress on account of goal to the optimal value of the function is achieved .
Quoted practical example based on the simplex method efficiency approved and primary and hesitant issues solutions between compatibility two one-sided optimality to the theory complete suitable arrival This is the case . of the method theoretical basics correctness again one there is proves .
Conclusion as in other words , the simples method linear programming issues in solution not only theoretical in terms of based on , maybe practical issues modeling and also widely used in optimization application possible universal method that is obtained . results economic planning , resources distribution and engineering issues in solution this from the method effective use opportunity gives .

5. References

1. Wentzel E.S. Issledovanie operation . — M.: Nauka, 1988.
2. Intriligator M. Mathematical optimization method and economic theory . — M.: Progress, 1985.
3. Dantzig G. Linear Programming and Extensions . — Princeton: Princeton University Press, 1963.
4. Bandi B. Basic linear programming . — M.: Radio and Svyaz, 1989.
5. Dzhemilev N.I., Eydelnant M.I. Sbornik zadach po lineinomu programmirovaniyu . — T.: Teacher , 1990.
6. Issledovanie operational / Pod ed. Moudera Dj., Elmagrabi S. — M.: Mir, 1985. T. I-II.