International Journal of Informatics and Data Science Research
ISSN 2997-3961 (Online)
Vol. 3, No. 2, Feb 2026,
Available online at: https://scientificbulletin.com/index.php/IJIDSR

Mathematical Models and Economic Analysis of Linear Programming Problems

[1] Khujaev I.K., [1,2] Hamdamov M.M., [2] Boborakhimova M.I., [1] Muzaffarov S.A, [1] Ibrayimov A.X.
[1] Oriental University
[2] Tashkent University of Information Technologies

Article information:
Manuscript received: 04 Dec 2025; **Accepted:** 10 Jan 2026; **Published:** 06 Feb 2026

Abstract: The article studies the role and importance of linear programming theory in solving problems of optimizing economic processes. The object of research is the problem of effective planning of production in conditions of limited resources. The article analyzes mathematical models of linear programming problems, the economic content of the objective function, and methods for forming a system of constraints. Also, the relationship between primal and dual problems is theoretically and practically substantiated.

The work presents the problem of optimal production planning based on a specific economic example, and its mathematical model is built. The range of possible solutions is determined, and the optimal solution is found using the geometric method. The results obtained are verified in the Python environment using the Simplex algorithm, and it is proven that they fully correspond to the solution using the graphical method. The results show that linear programming methods are an important tool for improving the efficiency of economic analysis, rational use of resources, and decision-making. The results of the study can be used in production management, economic planning, and optimal decision-making processes..

Keywords: linear programming, mathematical modeling, optimal planning, objective function, system of constraints, twin problem, economic analysis, Simplex algorithm.

1. Introduction

Nowadays, the issues of effective management of economic systems, rational use of resources, and optimal decision-making are becoming increasingly important. In solving such issues, mathematical modeling methods, in particular, the theory of linear programming, play an important role. Linear programming methods are widely used in the fields of economics, production, logistics, finance, and management, allowing us to determine the most optimal solutions in conditions of limited resources.

The basis of linear programming problems is an objective function and a system of constraints. The objective function expresses the main goal of the economic process - for example, maximizing profit or minimizing costs. The constraints are determined by available resources, production capacity, or other economic factors. In this regard, the mathematical model of linear programming problems serves to formally analyze economic processes [1,2].

One of the important concepts of linear programming theory is the dual problem. The dual problem is closely related to the primal problem and, from an economic point of view, allows us to determine the shadow prices of resources, assess the importance of constraints, and analyze the effectiveness of decisions. The interrelationship between solutions to the primal and dual problems serves to further deepen economic analysis.

This article analyzes the mathematical models of linear programming problems, the economic content of the objective function, and the theoretical and practical aspects of the twin problem. The obtained results can be used to optimize economic processes and increase the efficiency of decision-making.

2. Materials and methods

The problem of determining the most optimal production plan, taking into account the existing economic and material dependencies between production parameters, has led to the formation of a new class of problems called linear programming problems (LPPs). Linear programming problems are of great importance in optimizing production processes and allow for effective decision-making in conditions of limited resources.

In the process of building a mathematical model, a system of constraints is formed based on the available production resources, market prices, and production standards, and a function form called the objective function (OF) is selected [3,4]. The objective function expresses the main direction of the economic process and takes on different meanings depending on the formulation of the problem. If the problem is related to increasing income, it is required to determine the maximum value of the objective function. On the contrary, if the problem is aimed at reducing costs, the main goal is to find the minimum value of the objective function.

In most cases, the constraints that represent production resources, labor, and their capabilities are given in the form of linear functions. Also, as a result of the fact that the objective function is also expressed in linear form, such problems are called linear programming problems (LPPs). Here, the term "programming" is used in the sense of planning, that is, it is required to create an optimal production plan aimed at maximizing revenue or minimizing costs.

These problems are usually limited by the limitations of traditional optimization methods, which will be discussed in more detail in the following sections. Therefore, there are effective methods specifically designed to solve linear programming problems, and this article will introduce some of them.

In order to get an initial idea of linear programming problems, a simple economic problem related to production is considered below. It should be noted in advance that the production standards given in the problem have conditional values, and if necessary, it is not difficult to replace them with real economic indicators.

There is a small private enterprise that produces two types of fruit juices. The enterprise has 30 kg of cherries, 45 kg of apples, and 12 kg of sugar. To produce 1 can of juice of the first type, 0.1 kg of cherries, 0.5 kg of apples, and 0.1 kg of sugar are needed. To produce 1 can of juice of the second type, 0.3 kg of cherries, 0.2 kg of apples, and 0.1 kg of sugar are needed. The price of 1 can of juice of the first type is 1000 soums. The price of 1 can of juice of the second type is 1400 soums. Determine the optimal production plan, that is, determine the number of juices of the first and second types that should be produced in order to maximize the enterprise's income.

The economic statement of the issue is expressed as follows. It is considered a sufficiently complex and serious problem, and a private entrepreneur who solves this issue correctly and reasonably will have the opportunity to increase the efficiency of his enterprise, establish stable production and ensure the long-term development of the enterprise.

In the conditions of a modern market economy, the rapid change in the economic environment requires constant improvement of the planning process. In this regard, the algorithmization of planning becomes an integral and continuous part of the production management system. This approach is of great importance for the efficient use of production resources, reducing economic risks, and ensuring competitiveness.

3. Results and discussion

We begin to build a mathematical model of the above economic problem. x_1, x_2 We determine the unknown number of cans of the first and second types of fruit juices produced at the enterprise. In this case, the production plan must correspond to the production resources available at the enterprise. To do this, the amount of resources spent on production should not exceed the corresponding resource reserve. Based on these requirements, we determine the resource consumption and the conditions for resource consumption in accordance with the existing standards (cherries, apples and sugar):

$$\begin{cases} 0,1x_1 + 0,3x_2 \le 30 \\ 0,5x_1 + 0,2x_2 \le 45 \\ 0,1x_1 + 0,1x_2 \le 12 \end{cases} \qquad (1)$$

$$x_1 \ge 0;\ x_2 \ge 0. \qquad (2)$$

Here, the enterprise's revenue is expressed by the following objective function :

$$L\left(x_1, x_2\right) = 1000x_1 + 1400x_2 \to \max \qquad (3)$$

Thus, a mathematical model and objective function of the above economic problem were constructed. The constructed model fully takes into account all the information given in the problem conditions, and the objective function represents the enterprise's income. This example, expressed by formulas (1)–(3), is one of the simplest forms of a linear programming problem (LPP).

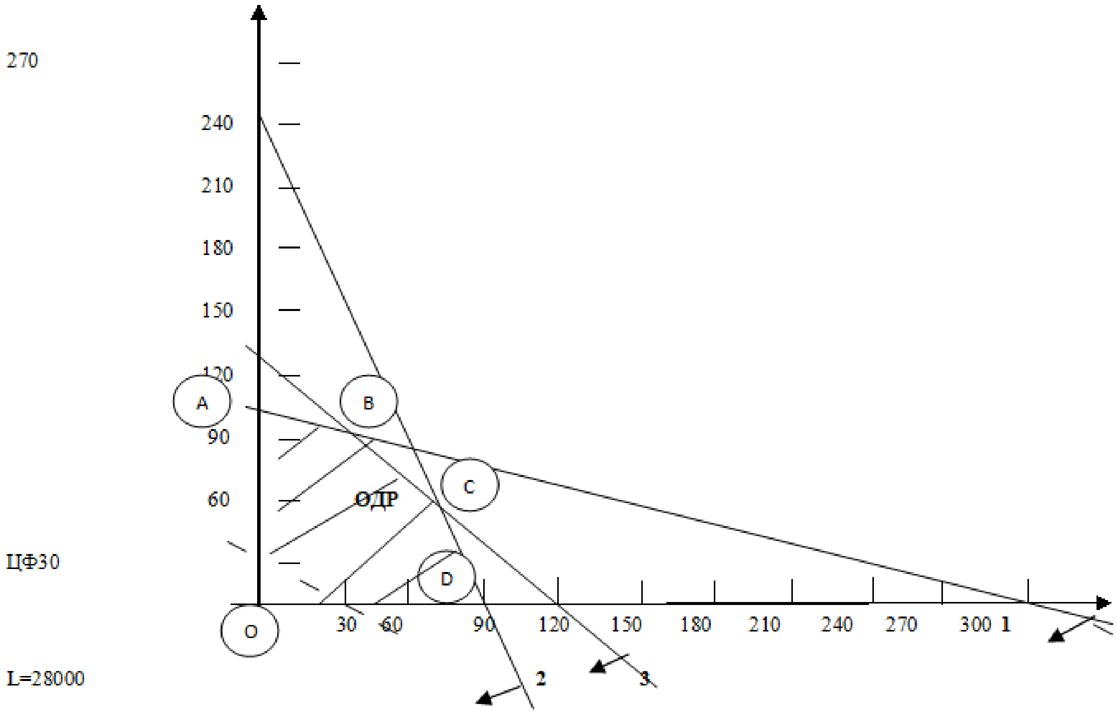
The content of the mathematical problem is to determine the maximum value of the objective function (3) among the points satisfying conditions (1)–(2) in the coordinate plane OX_1X_2. The set of points satisfying conditions (1)–(2) is called the feasible solution domain (FSD). The coordinates of any point in the FSD represent a feasible production plan. Therefore, the number of feasible plans is infinitely large. The problem of extracting the optimal plan from this set is nontrivial.

We will get acquainted with one of the methods for solving linear programming problems that can be used in two and three dimensions. It should be noted in advance that traditional methods for finding extrema, which are based on setting the first-order derivatives to zero, cannot be used in this case.

$$\frac{\partial L}{\partial x_1} = 1000 \ne 0;\ \frac{\partial L}{\partial x_2} = 1400 \ne 0$$

That is, there are no stationary points.

Geometric method of solving ChDM. Let us consider the area of the OX_1X_2 plane corresponding to conditions (1)-(2). Each of the conditions (1) separates a certain half-plane, and the conditions (2) separate the first quarter of the coordinate plane [5-8].



1 – Figure. Geometric method

Thus, we obtain the MBES, that is, the convex pentagon OABCD is formed (Figure 1). $L = 28000.$ The graph of the objective function is also given in this figure. It is clear that L as the value of ni increases, the MF increases. We need the maximum value of MF. As can be seen from Figure 3, such an increase is possible until it leaves the MBES, that is, until there is at least one possible solution on this straight line. Usually, the final intersection point of the MF and MBES is at one of the vertices of the MBES. Therefore, the vertices of the MBES polygon are the set for which we need to find the optimal solution. These polygon vertices, or rather their coordinates, are called reference solutions (TS). Thus, the solution algorithm comes down to finding a reference solution. In our case, the coordinates of the points A, B, C, D are easily found: A(0;100), B(70;50), C(30;90), D(90;0). At these points, we find the objective function values :

$$L_A = 140000; L_B = 140000; L_C = 156000; L_D = 90000.$$

As can be seen from the obtained values, the optimal plan is at point S and $x_1 = 30; x_2 = 90.$ thus , the profit is maximum when 30 cans of the first type of juice and 90 cans of the second type of juice are produced.

It should be noted here that when the spending norm, resource reserve, and market price change, only the corresponding coefficients in expressions (1)-(3) change , while the solution algorithm does not change.

The above problem allows us to construct a general mathematical model of such types of MNEs:

$$\begin{cases} \sum_{j=1}^{n} a_{ij} x_j \le b_i, \quad i = 1, 2, ..., m, & (4) \\ x_j \ge 0, \quad j = 1, 2, ..., n, & (5) \\ L(x) = \sum_{j=1}^{n} c_j x_j \to \max. & (6) \end{cases}$$

If we translate the conditions of the problem (4)-(6) into the language of economics, we can form the following example.

Enterprise m based on various resources produces a product of type j. The resource stock is equal to b_1 ,b_2 ,…,b_m., respectively. The i -th resource cost for the production of one product of type j is a_{ij} unit. The price of one product of type j is c_j unit of money. Determine the optimal production plan that maximizes the enterprise's profit.

It should be noted that for the issue to have a complete and realistic economic content, it is necessary to take into account, along with production resources, energy, transport and labor resources. These factors directly affect the production process and increase the accuracy and practical significance of the mathematical model.

As the values of the parameters m and nnn increase, the problem expressed by formulas (4)–(6) also becomes more complicated, and its solution requires much more work and computational resources. In such cases, it is necessary to resort to software calculation methods to find the optimal solution. In the following sections, we will get acquainted with one of such methods in detail.

First, let's touch on another feature of ChDM. Optionally, a twin problem can be constructed to the CDM of the form (4)-(6). It looks like this:

$$\begin{cases} \sum_{i=1}^{m} a_{ij} y_i \ge c_j, \quad j = 1, 2, ..., n, & (7) \\ y_i \ge 0, \quad i = 1, 2, ..., m, & (8) \\ Q(y) = \sum_{i=1}^{m} b_i y_i \to \min. & (9) \end{cases}$$

The theorem, known as the twin theorem, states that if problems (4)-(6) can be solved, then the twin problems (7)-(9) can also be solved, and their optimal values are equal:

$$L_{\max} = Q_{\min}.$$

Solution the existence determination and taken solution correctness in evaluation this from the condition wide is used . Some in cases twin (dual) issue to the main (primal) issue relatively simpler to appear owner to be possible . Such in situations problem solution and the result to check twin from the matter start to the goal suitable is considered .

Problem solving in Python the code is as follows:

```
imported pulp

# Let's create a problem (maximization)

model = pulp.LpProblem("Juice_production", pulp.LpMaximize)

# Variables
```

```
x = pulp.LpVariable('x', lowBound=0) # Type 1 juice
y = pulp.LpVariable('y', lowBound=0) # Type 2 juice
# Objective function (income)
model += 1000*x + 1400*y
# Restrictions
model += 0.1*x + 0.3*y <= 30 # cherry
model += 0.5*x + 0.2*y <= 45 # apple
model += 0.1*x + 0.1*y <= 12 # sugar
# We will solve the problem
model.solve()
# We will output the results
print("Status:", pulp.LpStatus[model.status])
print("Number of juice types 1 =", pulp.value(x))
print("Number of juice types 2 =", pulp.value(y))
print("Maximum return =", pulp.value(model.objective))
```

The following results were obtained through this software tool:

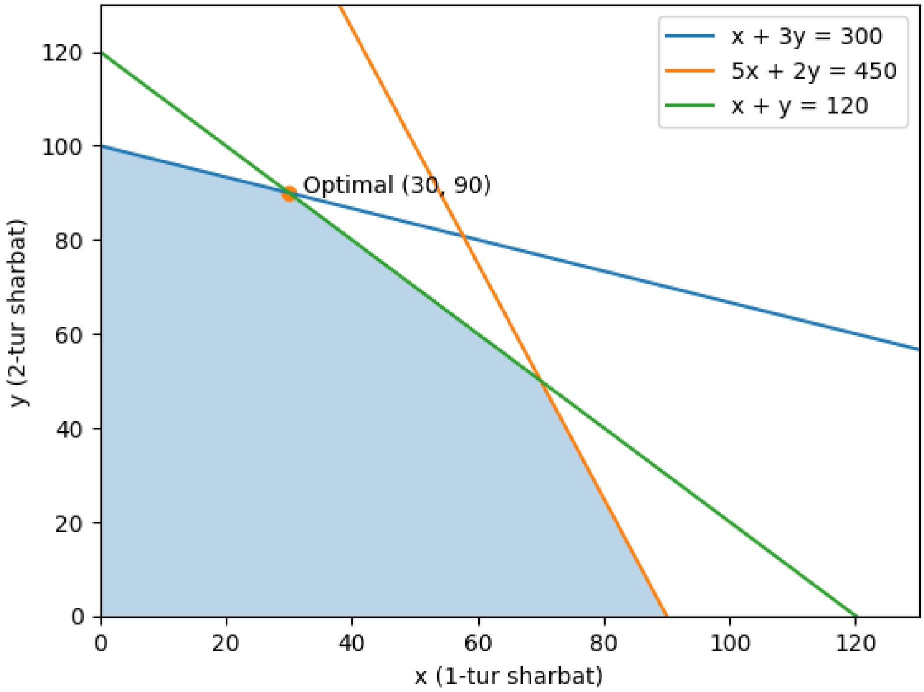


Figure 2. A linear programming problem.

The given diagram shows a graphical solution to the problem of optimal production planning. In the coordinate plane, the number of juices of the first type is located along the xxx axis, and the number of juices of the second type is located along the yyy axis. The constraints are given as straight lines corresponding to the amount of resources, which represent the reserves of cherries, apples, and sugar, respectively.

The set of points that satisfy all the constraints forms the feasible solution domain (FDS). This domain is depicted as a shaded polygon in the diagram, and any point located in this region represents a feasible production plan. However, the FDS is a bounded domain, that is, the production volumes are strictly limited by resources.

According to the theory of linear programming, the extreme value of the objective function is obtained at the corner points of the possible solutions. Therefore, in the process of graphical analysis, the main corner points of the MBES were determined, and the value of the objective function was calculated at each point. As a result, it was found that the objective function has the largest value at the point with coordinates (30; 90).

This point on the graph shows that the enterprise's profit is maximized when 30 of the first type of juice and 90 of the second type are produced. In this case, the use of resources is relatively balanced, and it can be seen that the resources of sugar and cherries in particular are fully or almost fully used. Therefore, this production plan is the most efficient given the available resources.

Also, the result obtained using the graphical method fully agrees with the solution obtained using the Simplex algorithm in the Python environment. This once again confirms the correctness of the constructed mathematical model and the optimality of the obtained solution.

The economic content of the twin problem is explained by determining the optimal, that is, marginal, prices of production resources. These prices allow us to assess their economic value in conditions of limited resources and are of great importance in analyzing the effectiveness of production decisions being made.

4. Conclusion

In this work, the theoretical foundations of linear programming problems related to optimal planning of production and their economic content were considered. It was proved that the use of mathematical modeling methods is important for the effective organization of production processes in conditions of limited resources. In particular, the possibility of formalizing the economic problem through the objective function and the system of constraints was demonstrated.

In the course of work, a linear programming model was built on the example of a practical economic problem related to production, the field of possible solutions was determined, and the optimal production plan was found using a graphical method. The obtained results clearly showed the effectiveness of linear programming methods in making decisions aimed at maximizing the company's income.

The research results show that traditional extremum detection methods are not applicable for linear programming problems, which requires the development and application of special solution methods. The use of linear programming methods is an important tool for planning production, rational use of resources and increasing economic efficiency in the modern market economy.

The obtained results can be widely used in economic analysis, production management and decision-making processes, and their implementation in practical activities will serve to increase the competitiveness of enterprises.

5. References

1. Akulich, I.A. (1986). Mathematical Programming in Examples and Problems: A Textbook for Universities. Moscow: Vysshaya Shkola. (in Russian)

2. Bandi, B. (1989). Fundamentals of Linear Programming. Moscow: Radio i Svyaz. (in Russian)

3. Venttsel, E.S. (1988). Operations Research: Problems, Principles, Methodology. Moscow: Nauka. (in Russian)
4. Dzhemilev, N.I., & Eidel'nant, M.I. (1990). Collection of Problems on Linear Programming. Tashkent: Uqituvchi. (in Russian)
5. Zamkov, O.O., et al. (1999). Mathematical Methods in Economics. Moscow: DIS. (in Russian)
6. Intriligator, M. (1985). Mathematical Optimization and Economic Theory. Moscow: Progress. (Translated from English)
7. Moudder, J., & Elmaghraby, S. (Eds.). (1985). Operations Research (Vols. I–II). Moscow: Mir. (in Russian)
8. Kremer, N.Sh. (Ed.). (1997). Operations Research in Economics. Moscow: UNITY. (in Russian)